WAKISSHA JOINT MOCK EXAMINATIONS
MARKING GUIDE
Uganda Advanced Certificate of Education
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MATHEMATICS P425/1

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1. $x^2 + 2x + \frac{12}{x^2+2x} = 7$

Let $Z = x^2 + 2x$

$Z + \frac{12}{Z} = 7$

$Z^2 - 7Z + 12 = 0$ M1

$(Z - 4)\ (Z - 3) = 0$ M1

Either $Z - 4 = 0$ or $Z - 3 = 0$

$Z = 4$ or $Z = 3$

Either $x^2 + 2x = 4$ or $x^2 + 2x = 3$

$x^2 + 2x - 4 = 0$ or $x^2 + 2x - 3 = 0$ M1

$x = \frac{-2 \pm \sqrt{4 - 4.1.^{-}4}}{2 \times 1}$ $(x + 3)\ (x - 1) = 0$

$x = \frac{-2 \pm \sqrt{20}}{2}$ Either $x + 3 = 0$ or $x - 1 = 0$ A1

$x = \frac{-2 \pm 2\sqrt{5}}{2}$ $x = -3$ or $x = 1$

$x = -1 \pm \sqrt{5}$

$\therefore x = -3,\ 1,\ \ -1 \pm \sqrt{5}$ B1

OR

$(x+2x)^2 - 7(x^2+2x) + 12 = 0$

$-4\ -3$ B1

$(x^2+2x)^2 - 4(x^2+2x) - 3(x^2+2x) + 12 = 0.$

$(x^2+2x)[(x^2+2x) - 4] - 3[(x^2+2x) + 4] = 0$

$(x^2+2x-3)(x^2+2x-4) = 0.$ M1

Either $x^2+2x-3 = 0$

$3\ -1$

$x = -3 \quad x = 1$

or $x^2+2x-4 = 0.$

$x = -1 \pm \sqrt{5}$ A1

B1

05

2. Let $x = \tan u$

$\frac{dx}{du} = \sec^2 u$ M1

$dx = \sec^2 u\ du$

$= \int \frac{\sec^2 u}{(\tan^2 u + 1)^2}\ du$

$= \int \frac{1}{\sec^2 u} du$

$= \int \cos^2 u\ du$

but $\cos^2 u = \frac{1}{2}[1 + \cos 2u]$

$= \frac{1}{2}\int (1 + \cos 2u) du$ A1

$= \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

$= \left[\frac{1}{2}\tan^{-1} x + \frac{1}{4}\sin 2(\tan^{-1} x\right]_0^1$ M1

$= \left(\frac{1}{2}\ tan_1^{-1} + \frac{1}{4}\sin 2\ (tan_1^{-1}\right) - \left(\frac{1}{2}\ tan^{-1}0 + \frac{1}{4}\sin 2(tan^{-1}0\right)$ M1

$= 0.642699081 - 0$

$= 0.642699081 \doteq 0.6427$ (4dp) A_1

05

3. Displacement of $2x - 3y - z + 1 = 0$ from the ~~unique~~ origin is $d_1 = \frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}$

$d_1 = \frac{2(0) - 3(0) + -1(0) + 1}{\sqrt{4+9+1}}$ M_1

$d_1 = \frac{1}{\sqrt{14}} = \frac{\sqrt{14}}{14}$

$d_1 = \sqrt{\frac{14}{14}}$ $= \checkmark$ A_1

$2x-3y-z+1=0$
Rearrange
$2x-3y-z=-1$ M1
Comparing
$ax+by+cz=d.$ $d_1=-1$ A1

$6x-9y-3z=5$ M1
Comparing
$ax+by+cz=d$ $d_2=5$ A1

Displacement of $6x$ -9y-3z -5 from the origin is $d_2 = \frac{6(0)-9(0)-3(0)-5}{\sqrt{36+81+9}}$ M_1

$d_2 = \frac{-5}{3\sqrt{14}}$ A_1

$d_2 = \frac{-5\sqrt{14}}{42}$

The planes are of opposite sides of the origin because their displacements have opposite sides. A1

05

The planes are at the opposite sides of the origin because their displacements have opposite signs. 05

A_1

4. a) Number of ways $= \frac{10!}{4!}$

$= 151{,}200$ ways.

b) Number of arrangements with out I S' is 6 ! = 720 ways. M_1

The I's are to be placed in 7 spaces of which we need 4, Number of selections of the spaces for I S' is $7C_4$ and now if I S' are separated, Number of arrangements is $7C_4$ x 720 = 25,200 ways. A_1

05

AJ

5. Let $tan^{-1}\left(\frac{1-x}{1+x}\right) = A.$

$tan\ A = \frac{1-x}{1+x}$ $\qquad tan^{-1}(x) = B$ B1

$A = \frac{1}{2}B$ $\qquad tan\ B = x$

$2A = B$

$tan\ 2A = tan\ B$

$\frac{2\tan A}{1-tan^2A} = \tan B$

$\frac{2\left(\frac{1-x}{1+x}\right)}{2-\left(\frac{1-x}{1+x}\right)^2} = \frac{x}{1}$ M1

$$2\left(\frac{1-x}{1+x}\right)=x\left(1-\left(\frac{1-x}{1+x}\right)^2\right)$$ —— M1

$$2(1-x)(1+x)=x(1+x)^2-(1-x)^2$$

$$2(1-x^2)=x(1+2x+x^2-1+2x-x^2)$$

$$2(1-x^2)=x(4x)$$

$$2-2x^2=4x^2$$ —— A1 (Answer $x=\pm\frac{1}{\sqrt3}$.

$$x^2=\frac{1}{3}$$

$$x=\frac{1}{\sqrt3}\ \cancel{=}\ -\frac{1}{\sqrt3}$$

Testing

$$\tan^{-1}\left(\frac{1+\frac{1}{\sqrt3}}{1-\frac{1}{\sqrt3}}\right)=\frac{1}{2}\tan^{-1}\left(\frac{-1}{\sqrt3}\right)$$

$$75=\frac{1}{2}(-30)$$

$$\therefore x=+\frac{1}{\sqrt3}$$ B1

6. AJ	$x^2+y^2-6x+8=0$ and $x^2+y^2-2x-2y=7$ The equation to any circle which passes through the origin is $x^2+y^2+2gx+2fy=0$	B1
	This circle cuts the first of the given circles orthogonally if $2g(-3)+2f(0)=8$ ie $1t-6g=8$ or $g=-4/3$	B1
	If cuts the second of the given circle orthogonally if $2g(-1)+2f(-1)=-7$ This gives $f=\frac{7}{2}-g=\frac{7}{2}+\frac{4}{3}=\frac{29}{6}$ and the required equation to the circle is $x^2+y^2-\frac{8}{3}x+\frac{29}{3}y=0$	B1
	or $3x^2+3y^2-8x+29y=0$	M1
		~~A1~~
		(05)

⇒ Supposed to get the centre and c. → or substitute the From radius into the expression.

$x^2+y^2+2gx+2fy+c=0$

$x^2+y^2-\frac{8}{3}x-\frac{29}{3}=0$

$3x^2+3y^2-8x+29=0$ A1

For $x^2+y^2+2gx+2fy+c=0$ at $(0,0)$ $c=0$.

$x^2+y^2-6x+8=0$

$(x-3)^2+(y-0)^2-9+8=0$

$(x-3)^2+(y-0)^2-1=0$ M1

Centre $(3,0)$ $c=-1$.

$9+6g-1=0$

$9+6g=1$

$g=\frac{-8}{6}=-4/3$ A1

$-\frac{8}{3}+2f=7$

$-8+6f=21$

$6f=29$

$f=29/6$.

$x^2+y^2-2x-2y=7$

$(x-1)^2-1+(y-1)^2-1=7$ M1

$(x-1)(y-1)^2-9=0$

Centre $(1,1)$ $c=-9$

$1+1+2g+2f-9=0$

$2g+2f-7=0$ A1

$2\left(\frac{-4}{3}\right)+2f=7$

7.	Let $y = \frac{(x+1)^2(x+2)}{(x+3)^3}$ By using the quotient role Let $u = (x+1)^2(x+2)$ and $v = (x+3)^3$ B1 (correct u and correct v) $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $\frac{du}{dx} = 2(x+1)(x+2)+(x+1)^2$ M1 (defferetiating u and v) $=(x+1)(2x+4+x+1)$ $=(x+1)(3x+5)$ $\frac{dv}{dx} = 3(x+3)^2 - 1 = 3(x+3)^2$ M1 (substiting for du and dv) $\frac{dy}{dx} = \frac{(x+3)^3(x+1)(3x+5)-(x+1)^2(x+2(x+3)^2}{((x+3)^3)^2}$ $=\frac{(x+1)[(3x^2+14x+15)-(3x^2+9x+6)]}{(x+3)^4}$ M1 (simplyfying the values) $\frac{dy}{dx} = \frac{(x+1)(5x+9)}{(x+3)^4}$ A1 (Ans)	B1 M1 for $\frac{dy}{dx}$ M1 for Substitution M1 for Simplifying A1 (05)

05

let $y = \frac{(x+1)^2(x+2)}{(x+3)^3}$

$\ln y = \ln\left(\frac{(x+1)^2(x+2)}{(x+3)^2}\right)$

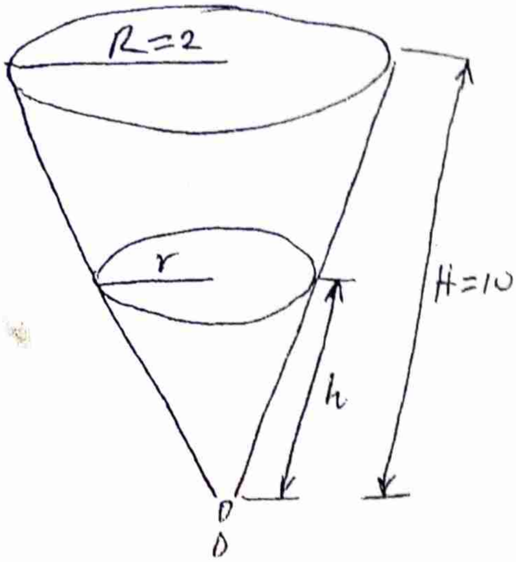
$\ln y = 2\ln(x+1) + \ln(x+2) - 2\ln(x+3)$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x+1} + \frac{1}{x+2} - \frac{2}{x+3}$

$\frac{dy}{dx} = \frac{2(x+2)(x+3)+(x+1)(x+3)-2(x+1)(x+2)}{(x+1)(x+2)(x+3)}\cdot y$

$\frac{dy}{dx} = \frac{(x+1)(5x+9)}{(x+3)^4}.$

8.



$$\frac{r}{h}=\frac{R}{H}$$

$$r=\frac{R}{H}h$$

$$=\frac{2}{10}h$$

$$\therefore r=\frac{1}{5}h$$

$$V=\frac{1}{3}\pi\left(\frac{1}{5}h\right)^2 h$$

M₁ for expressing r ratio of h

$$=\frac{1}{75}\pi h^3$$

$$\frac{dv}{dh}=\frac{\pi}{25}h^2$$

M₁ for $\frac{dv}{dh}$

Given $\frac{dv}{dt}=\frac{-\pi}{100}cm^3s^{-1}$; negative indicates decrease in volume of sand.

$$\frac{dh}{dt}=\frac{dv}{dt}\div\frac{dv}{dh}=\frac{-\pi}{100}\times\frac{25}{\pi h^2}=\frac{-1}{4h^2}$$

M₁ for get $\frac{dh}{dt}$

When h=5cm;

$$\frac{dh}{dt}=\frac{-1}{4(5)^2}$$

M₁ for substituting for h

$=-0.01cms^{-1}$ is decreasing at rate of 0.01cms^{-1}

A₁

(05)

SECTION B (60 MARKS)

9. (a)

$$\text{Sin}\left(3\theta-\frac{\pi}{2}\right)+\sin 4\theta=0$$

$$\Rightarrow 2\sin\left(\frac{4\theta+3\theta-\frac{\pi}{2}}{2}\right)\cos\left(\frac{4\theta-3\theta+\frac{\pi}{2}}{2}\right)=0$$

M₁ for using factor formula

Either $\sin\left(\frac{7\theta}{2}-\frac{\pi}{4}\right)=0$ or $\cos\left(\frac{\theta}{2}+\frac{\pi}{4}\right)=0$

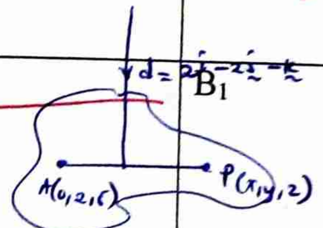
A₁ for both

For $\cos\left(\frac{\theta}{2}+\frac{\pi}{4}\right)=0$

	$\frac{\theta}{2}+\frac{\pi}{4}=\frac{\pi}{2}$ $\theta=\frac{\pi}{2}$ (A₀ − Reading in degrees) For $\sin\left(\frac{7\theta}{2}-\frac{\pi}{4}\right)=0$ $\left(\frac{7\theta}{2}-\frac{\pi}{4}\right)=0,\pi,2\pi,3\pi,{}^{-}\pi,{}^{-}2\pi$ $\frac{7\theta}{2}=\frac{\pi}{4},\frac{5\pi}{4},\frac{9\pi}{4},\frac{13\pi}{4},\frac{-3\pi}{4},\frac{-7\pi}{4},$ (A₀ − Reading in degrees) $\theta=\frac{\pi}{14},\frac{5\pi}{14},\frac{-3\pi}{14},\frac{-\pi}{2},\ \pi/2.$ $\theta=\ -\pi/2,\ \frac{-3\pi}{14},\ \pi/2,\ \pi/14,\ \frac{5}{14}\pi$	M_1 for reading angle A1 for $\theta=\frac{\pi}{2}$ M_1 for reading angles A_1 for all the 4 angles **06**
(b)	$\sec^2\theta=\frac{1}{\cos^2\theta}$ Show + Left to Right But $\cos^2\theta=\frac{1}{2}(\cos 2\theta+1)$ $\sec^2\theta=\frac{1}{\frac{1}{2}(\cos 2\theta+1)}$ $=\frac{2}{\cos 2\theta+1}$ $=\frac{\frac{2}{\cos 2\theta}}{\frac{\cos 2\theta}{\cos 2\theta}+\frac{1}{\cos 2\theta}}$ $\sec^2\theta=\frac{2\sec 2\theta}{1+\sec 2\theta}$ $\sec^2\theta=\frac{2\sec 2\theta}{1+\sec 2\theta}$ $\sec^2\theta=\frac{2-2+2\sec 2\theta}{1+\sec 2\theta}$ $\sec^2\theta=\frac{2+2\sec 2\theta-2}{1+\sec 2\theta}$	M_1 M_1 B_1 M_1

	$=\frac{2(1+\sec 2\theta)}{1+\sec 2\theta}-\frac{2}{1+\sec 2\theta}$	M_1
	$\sec^2\theta = 2-\frac{2}{1+\sec 2\theta}$	B_1 ~~(06)~~
10. (a)	$\left(\frac{1+3x}{2-x}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}}(1+3x)^{\frac{1}{2}}\left(1-\frac{x}{2}\right)^{\frac{-1}{2}}$	
	$(1+3x)^{\frac{1}{2}} = 1+\frac{1}{2}(3x)+\frac{1}{2}\left(\frac{-1}{2}\right)\frac{\left(\frac{3x}{[redacted]}\right)^2}{2!}+\ldots\ldots$	
	$=1+\frac{3}{2}x-\frac{9}{8}x^2+---$	M_1
	$\left(1-\frac{x}{2}\right)^{\frac{-1}{2}} = 1-\frac{1}{2}\left(\frac{-x}{2}\right)+\frac{-1}{2}\left(\frac{-3}{2}\right)\frac{\left(\frac{-x}{2}\right)^2}{2!}+\ldots\ldots\ldots\ldots$	
	$=1+\frac{x}{4}+\frac{3}{32}x^2+----$	M_1
	$(1+3x)^{\frac{1}{2}}(2-x)^{\frac{-1}{2}} = \frac{1}{\sqrt{2}}\left(1+\frac{3}{2}x-\frac{9}{8}x^2\right)\left(1+\frac{x}{4}+\frac{3}{32}x^2\right)$ M1	M1 (attempting to multiply)
	$=\frac{1}{\sqrt{2}}+\frac{7}{4\sqrt{2}}x-\frac{21x^2}{32\sqrt{2}}$ M1	M_1 (Correct expression)
	$\left(\frac{1+\frac{3}{5}}{2-\frac{1}{5}}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}}+\frac{7}{4\sqrt{2}}\left(\frac{1}{5}\right)-\frac{21}{32\sqrt{2}}\left(\frac{1}{5}\right)^2$ A1	A_1 (substitution)
	$\frac{\sqrt{8}}{3} = 0.707106781+0.247487373-0.0[redacted]61553$	
	$\sqrt{8} = 3(0.936032601)$	~~M_1~~
	$=2.8080978$	A_1
	$\approx 2.8\,(2sig.figs)$	B_1
		(07) 07

(b)	$\left[\sqrt{(3-x)}-\sqrt{(7+x)}\right]^2=\left[\sqrt{(16+2x)}\right]^2$	M_1 for squaring
	$3-x+7+x-2\sqrt{(3-x)(7+x)}=16+2x$	
	$-2\sqrt{(3-x)(7+x)}=6x+2x$	
	$\left[-(3-x)(7+x)\right]^2=[3+x]^2$	
	$(3-x)(7+x)=9+x^2+6x$	
	$21-4x-x^2=9+x^2+6x$	
	$2x^2+10x-12=0$	
	$x^2+5x-6=0$	
	$(x+6)(x-1)=0$	M_1 for method solving
	$x+6=0$ or $x-1=0$	A_1 for both values of x
	$x=-6$ or $x=1$	
	Checking	
	$\sqrt{(3+6)}-\sqrt{(1)}=\sqrt{4}$ Verification is part and partial of using this method.	M_1 for checking
	$\sqrt{9}-\sqrt{1}=\sqrt{4}$	
	$3-1=2$	
	When $x=1$	B_1
	$\sqrt{2}-\sqrt{8}\neq\sqrt{18}$	(05) 05
	$\therefore x=-6$	
11. (a)	Direction vector of the line is $\underset{\sim}{d}=\begin{pmatrix}2\\-2\\-1\end{pmatrix}$ B_1	B_1
	This is the normal to the plane since $\underline{r}.\underline{n}=\underline{n}.\underline{a}$	B_1 — shortage of marks.
	$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}2\\-2\\-1\end{pmatrix}=\begin{pmatrix}2\\-2\\-1\end{pmatrix}.\begin{pmatrix}0\\2\\5\end{pmatrix}$ $\underset{\sim}{d}.\overrightarrow{PA}=0.$	M_1
	$2x-2y-z=0-4-5$	
	$2x-2y-z+9=0$	M_1
		A_1
		(04) 04



(b)	Let $\frac{x-3}{2}=\frac{2-y}{2}=2-z=\lambda$ at the plane $\Rightarrow x=3+2\lambda, y=-2\lambda+2, z=2-\lambda$ still parametric equation of line putting x, y, z in π $2(3+2\lambda)+-2(-2\lambda+2)-(2-\lambda)+9=0$ $6+4\lambda+4\lambda-4-2+\lambda+9=0$ $9\lambda+9=0$ $\lambda=-1$ $\Rightarrow x=1, y=4$ and $z=3$ $x=3-2=1$, $y=+2+2=4$, $z=2+1=3$ $\therefore c\,(1,4,3)$	M1 (substituting the value of x, y, z) A1 (value of λ) M1 (for substituting for λ in x, y and z) A1 (point of intersection form) (04)
(c)	$\underset{\sim}{AC}=\underset{\sim}{OC}-\underset{\sim}{OA}$ $\underset{\sim}{AC}=\begin{pmatrix}1\\4\\3\end{pmatrix}-\begin{pmatrix}0\\2\\5\end{pmatrix}=\begin{pmatrix}1\\2\\-2\end{pmatrix}$ $\underset{\sim}{BC}=\underset{\sim}{OC}-\underset{\sim}{OB}$ $\underset{\sim}{BC}=\begin{pmatrix}1\\4\\3\end{pmatrix}-\begin{pmatrix}-1\\3\\1\end{pmatrix}=\begin{pmatrix}2\\1\\2\end{pmatrix}$ $\underset{\sim}{AC}.\underset{\sim}{BC}=\lvert\underset{\sim}{AC}\rvert\lvert\underset{\sim}{BC}\rvert\cos A\hat{C}B$ $A\hat{C}B=\cos^{-1}\left[\frac{\begin{pmatrix}1\\2\\-2\end{pmatrix}.\begin{pmatrix}2\\1\\2\end{pmatrix}}{\sqrt{9}\times\sqrt{10}}\right]$ $A\hat{C}B=\cos^{-1}0$ $A\hat{C}B=90^o$	B1 (obtaining AC) B1 (obtaining BC) M1 (substituting as solving) A1 (ans 90°) (04)

11. (a)	Direction vector of the line i.e $\underset{\sim}{d}=\begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$ This is the normal to the plane since $\underset{\sim}{r}.\underset{\sim}{n} = \underset{\sim}{n}.\underset{\sim}{a}$ $\begin{pmatrix} x \\ -y \\ -z \end{pmatrix} . \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} . \begin{pmatrix} 6 \\ 2 \\ 5 \end{pmatrix}$ $2x - 2y - z = 0 - 4 - 5$ $2x - 2y - z + 9 = 0$	B_1 M_1 M_1 A_1 **04 Marks**
b)	Let $\frac{x-3}{2} = \frac{2-y=2-z}{2} = \lambda$ at the plane $x = 3 + 2\lambda$, $y = 2\lambda$ $y = 2\lambda + 2$, $Z = 2 - \lambda$ putting x,y,z in II $2(3+2\lambda) + - 2(-2\lambda+2) - (2-\lambda) + 9 = 0$ $6 + 4\lambda + 4\lambda - 4 - 2 + \lambda + 9 = 0$ $9\lambda + 9 = 0$ $\lambda = -1$ $x = 1$, $y = 4$ and $Z = 3$ $\therefore c(1, 4, 3)$	
c)	$\underset{\sim}{AC} = \underset{\sim}{OC} - \underset{\sim}{OA}$ $\underset{\sim}{AC} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 6 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $\underset{\sim}{BC} = \underset{\sim}{OC} - \underset{\sim}{OB}$ $\underset{\sim}{BC} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$	

13 b	$\underset{\sim}{AC} \cdot \underset{\sim}{BC} = \lvert\underset{\sim}{AC}\rvert \, \lvert\underset{\sim}{BC}\rvert \cos A\hat{C}B$ $A\hat{C}B = \cos^{-1}\left[\dfrac{\begin{pmatrix}1\\2\\-2\end{pmatrix}\cdot\begin{pmatrix}2\\1\\2\end{pmatrix}}{\sqrt{10} \times \sqrt{10}}\right]$ $A\hat{C}B = \cos^{-1} 0$ $A\hat{C}B = 90^0$	$(x+2)^2 + 9^2$ $(x+2)^2 -$
12 a	$x^2 + 4x + 13 = x^2 + 4x + (2)^2 + 13 - (2)^2$ let $f(x) = (x+2)^2 + 9 = 0$ A1 $X = -2 \pm 3i$ $(x^2+4x+13 = (x+2+3i)(\cancel{x}+2-3i)$ M1 A1 Since the partial fractions have compl~~ex~~e denomitors ,we need to prepare for unknown numerators which are complex. $\dfrac{x+8}{x^2+4x+3} = \dfrac{x+8}{(x+2+3i)(x+2-3i)}$ $= \dfrac{A+Bi}{(x+2+3i)} + \dfrac{C+Di}{(x+2-3i)}$ $x + 8 = (A+Bi)(x+2-3i) + (C+Di)(x+2+3i)$ M1 $y\,x = -2 + 3i$ $-2+8+3i = (C+Di)(-2+3i+2+3i)$ $6+3i = (C+Di)6i$ $6+3i = 6Ci - 6D$ $6C = 3$ $C = ½$ $D = -1$ A1 If $x = -2 - 3i$ $-2+8-3i = (A+Bi)(-2-3i+2-3i)$ $6 - 3i = (A+Bi)(-6i)$	OR let $x = a \pm bi$ be the roots. The factors are $x - a - bi$ and $x - a + bi$ Multiply the factors and equate $(x-a-bi)(x-a+bi) = x^2+4x+13$. $x^2 - ax + b\cancel{xi} - ax + a^2 - a\cancel{bi} - b\cancel{xi} + a\cancel{bi} + b^2 = x^2+4x+13$ M1 $x^2 - 2ax - \cancel{bxi} + \cancel{bxi} + a^2 + b^2 = x^2+4x+13$ $x^2 - 2ax + a^2 + b^2 = x^2+4x+13$ $-2a = 4$ $a = -2$ $a^2 + b^2 = 13$ $4 + b^2 = 13$ $b = \sqrt{9}$ $b = \pm 3$.

The factors are
$x - 2 - 3i$ and
$x - 2 + 3i$ A1

$6 - 3i = -6Ai + 6B$

$-6A = {}^{-}3$

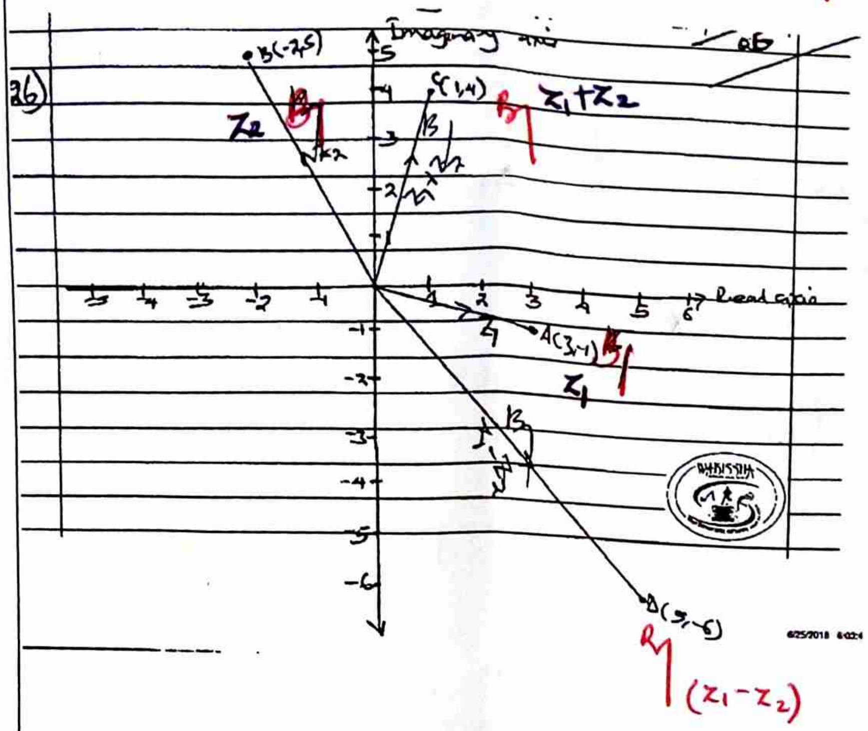
$A = \frac{1}{2}$

$B = 1$ A1

$$\frac{x+8}{x^2+4x+13} = \frac{\frac{1}{2}+i}{x+2+3i} + \frac{(\frac{1}{2} - i)}{(x+2-3i)}$$ A1

06

12 b (b)



Reading the values from the graph.

$|z_1 + z_2| = |1 + 4i| = \sqrt{1^2 + 4^2}$

$= \sqrt{17}$

$= 4.1231$ (4dp) B1

$\text{Arg}(z_1 + z_2) = \propto$

but $\propto = \tan^{-1}\left(\frac{4}{1}\right)$

$\text{Arg}(Z_1 + Z_2) = \tan^{-1}\left(\frac{4}{1}\right) = 75.96^0$ B1

$= 0.422\pi$

06

13 a

Let $y = \frac{x^2+2}{x^2-4}$

$yx^2 - 4y = x^2+2$

$(y-1)x^2 - 4y - 2 = 0$ M1

For real roots

$b^2 - 4ac \geq 0$

$0 - 4(y-1)(-4y-2) \geq 0$

$(y-1)(4y+2) \geq 0$

The critical values are $y = 1$ and $y = \frac{-1}{2}$ B1

	$y < \frac{-1}{2}$	$\frac{-1}{2} < y < 1$	$y > 1$
$(y-1)(4y+2)$	+	-	+

B1

$\frac{-1}{2} < y \cancel{<} 1$ from the table

$-\frac{1}{2} < y < 1$ from the table out

$y < \frac{-1}{2}$ or $y > 1$ from the table B1

for $(y-1)(4y+2) \geq 0$,

$y \leq \frac{-1}{2}$ or $y \geq 1$

for $y = 1$

$\frac{x^2+2}{x^2-4} = 1$

$x^2 + 2 = x^2 - 4$

$2 = -4$ which is impossible. (Discarded)

$\therefore$ Either $y \leq \frac{-1}{2}$ or $y > 1$

$\frac{-1}{2} \geq \frac{x^2+2}{x^2-4} > 1$ ~~as required~~ is the region B1

(13b) at $y = 1$, the turning point doesnot exist

for $y = \frac{-1}{2}$ the curve is maximum

$$\frac{-1}{2} = \frac{x^2+2}{x^2-4}$$ M1

$-x^2+4 = 2x^2 +4$

13 b $-3x^2 = 0$

$x^2 = 0$

$x = 0$

$\left(0, \frac{-1}{2}\right)$ is a maximum point A1

13 c) As f(x) I ∝, $x^2 - 4 = 0$

$x = \pm 2$ are the vertical asympototoes. B1

$f(x) = \frac{x^2 +2}{x^2-4}$

$f(x) = \frac{1+\frac{2}{x^2}}{1-\frac{4}{x^2}}$

As $x \pm \propto, \frac{2}{x^2}, \frac{4}{x_2} \rightarrow 0$

f(x) = 1

y = 1 is the horizontal a symptote B1

For intercepts ,if $x = 0$, $y = \frac{-1}{2}$

If y = 0, real x doesnot exist so the curve has no x intercepts.

Critical values are $x = \pm 2$

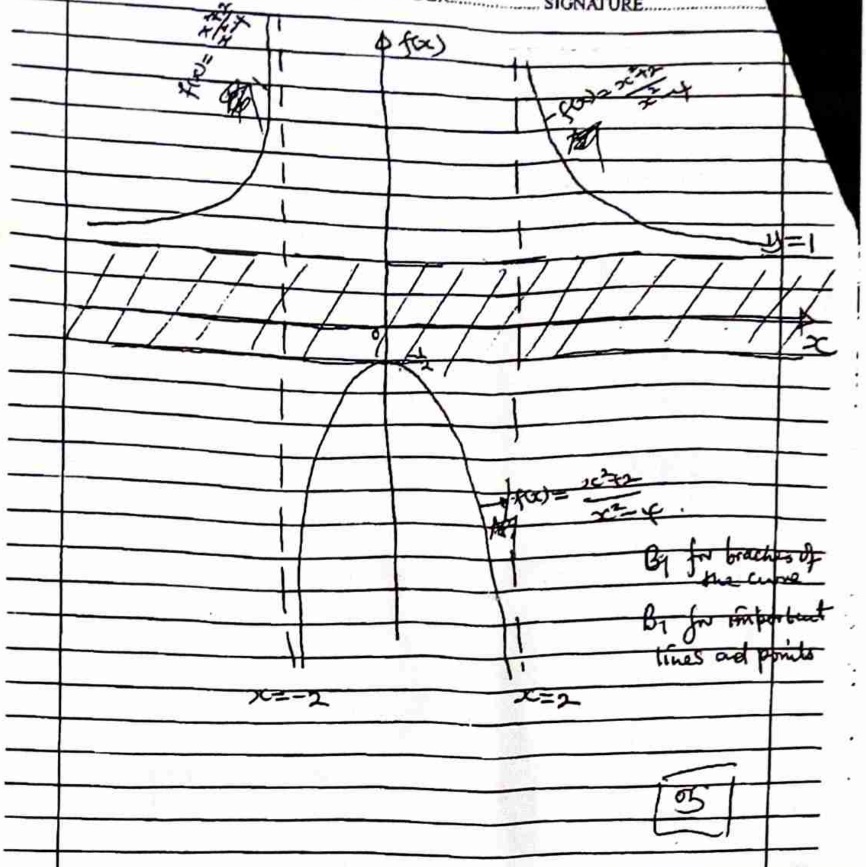
	$x<-2$	$-2<x<2$	$x>2$
x^2+2	+	+	+
x^2-4	+	–	+
$\frac{x^2+2}{x^2-4}$	+	-	+

B1

Do not write in this margin

CANDIDATE'S NAME:........................ INDEX No........

SUBJECT:........................ PAPER:........ SIGNATURE........



B1 — Branches of the curves

B1 — Important lines and asymptotes

14 a) At T, $x = ct$, $y = \frac{c}{t}$ — M1 — for differentiation

$\frac{dx}{dt} = c$, $\frac{dy}{dt} = \frac{-c}{t^2}$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= \frac{-c}{t^2} \cdot \frac{1}{c}$ — A1 — Gradient of tangent

$= \frac{-1}{t^2}$ is the gradient of the tangent at T

$m_1 x m_2 = -1$ for perpendicular lines

Gradient of the normal is $m_2 = t^2$ — B1 — Gradient of normal

$\frac{y-\frac{c}{t}}{x-ct} = t^2$

$y - \frac{c}{t} = t^2x - ct^3$ — M1 — Equation from t^2

$y = t^2x - ct^3 + \frac{c}{t}$

$y = t^2x + c\left(\frac{1}{t} - t^3\right)$. — A1 (correct equation of the normal)

b) Is the equation of the normal at T.

	Putting y = $t^2x+c\left(\frac{1}{t}-t^3\right)=c^2$	
	$x\left(t^2x+c\left(\frac{1}{t}-t^3\right)\right)=c^2$ ——	m1 (substituting the value of y)
	$t^2x^2+c\left(\frac{1}{t}-t^3\right)x=c^2$	
	$x^2+c\left(\frac{1}{t^3}-t\right)x=\frac{c^2}{t^2}$ ——	m1 (making quadratic equation)
	By completion of squares	
	$\left[x+\frac{c}{2}\left(\frac{1}{t^3}-t\right)\right]^2=\frac{c^2}{t^2}+\frac{c^2}{4}\left(\frac{1}{t^3}-t\right)^2$	
	$\left[x+\frac{c}{2}\left(\frac{1}{t^3}-t\right)\right]^2=\frac{c^2}{t^2}+\frac{c^2}{4t^6}-\frac{c^2}{2t^2}+\frac{c^2t^2}{4}$	
	$\left[x+\frac{c}{2}\left(\frac{1}{t^3}-t\right)\right]^2=c^2\left(\frac{1}{2t^3}+\frac{t}{2}\right)^2$	— m1 (solving the equation)
	$x+\frac{c}{2}\left(\frac{1}{t^3}-t\right)=\pm c\left(\frac{1}{2t^3}+\frac{t}{2}\right)$	
	$x=\frac{-c}{2}\left(\frac{1}{t^3}-t\right)\pm c\left(\frac{1}{2t^3}+\frac{t}{2}\right)$	
	Either $x=\frac{-c}{2}\left(\frac{1}{t^3}-t\right)+c\left(\frac{1}{2t^3}+\frac{t}{2}\right)$	
	x= ct	
	OR	

$x = \frac{-c}{2}\left(\frac{1}{t^3} - t\right) - \frac{c}{2}\left(\frac{1}{t^3} + t\right)$ M1

$x = \frac{-c}{t^3}$

$x = -ct^{-3}$ A1

since $xy = c^2$

$y = \frac{c^2}{-ct^{-3}}$

$y = -ct^3$ A1

$\therefore R$ is $(-ct^{-3}, -ct^3)$ B1

15 a) Let $t = \tan\frac{\theta}{2}$

$\frac{dt}{d\theta} = \frac{1}{2}\sec^2\frac{\theta}{2}$

$\frac{dt}{d\theta} = \frac{1}{2}(1+t^2)$ — M1

$d\theta = \frac{2dt}{1+t^2}$ — A1

change of limits

θ	o	$\frac{\pi}{2}$
t	o	1

— B1 (change of limits)

$\int_0^{\frac{\pi}{2}} \frac{5}{3\sin\theta + 4\cos\theta}\, d\theta = \int_0^1 \frac{5.\frac{2dt}{1+t^2}}{3\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right)}$

$= \int_0^1 \frac{5dt}{3t+2-2t^2}$

$= -\int_0^1 \frac{5dt}{2t^2 - 3t - 2}$

$= -\int_0^1 \frac{5dt}{2t^2 - 4t + t - 2}$

$= -\int_0^1 \frac{5dt}{2t(t-2) + (t-2)}$

$= -\int_0^1 \frac{5dt}{(t-2)(2t+1)}$

Let $\frac{5}{(t-2)(2t+1)} = \frac{A}{(t-2)} + \frac{B}{2t+1}$ M1

5 = A(2t+1)+B(t-2)

If t = 2

5 = 5A

A = 1 ✓

If t = $\frac{^-1}{2}$

$5 = \frac{^-5B}{2}$

B = -2 ✓ A1

$\rightarrow {}^-\int_0^1 \frac{5dt}{(t-2)(2t+1)} = {}^-\int_0^1 \left[\frac{1}{t-2} - \frac{2}{2t+1}\right] dt$ $\rightarrow \int_0^1 \left(\frac{2}{2t+1} - \frac{1}{t-2}\right) dt$.

$= {}^-In(t-2)\Big|_0^1 + In(2t+1)\Big|_0^1$ M1 $\rightarrow \int_0^1 \left(\frac{2}{2t+1} + \frac{1}{2-t}\right) dt$

$= -In|-1| + 1n|-2| + In3 - In1$

$= In\ 6$

$\cancel{d\theta} = 1.7918$

$\therefore \int_0^{\frac{\pi}{2}} \frac{5}{3\sin\theta + 4\cos\theta}$ $d\theta = 1.7918$ A1

07

15 b

Let $u = \text{In}x \quad \frac{du}{dx} = \frac{1}{x}$

$\frac{dv}{dx} = x^2 \qquad v = \frac{x^3}{3}$ B1 B1

$\int x^2 \text{In}\, x\, dx = \frac{x^3}{3} Inx - \int \frac{x^3}{3}, \frac{1}{x}\, dx$ M1 (substituting)

$= \frac{x^3}{3}\ Inx - \frac{1}{3}\int x^2 dx$

$= \frac{x^3}{3} Inx - \frac{1}{9}x^3 + c$ M1 (integrating)

$= \frac{x^3}{9}(3Inx - 1) + c$ A1

05

05

let T be the temp of the dog at any time, then R be the temp of the surrounding

16 a	$\frac{dT}{dt} \propto (T-R)$	m1
	$\frac{dT}{dt}$ = -K (T-R)	
	$\frac{dT}{(T-R)}$ = -Kdt	
	$\int \frac{dT}{(T-21)} = \int -kdt$	m1
	In(T-21) = -Kt + C	
	When t = 0 T = 39 °c	
	In(39-21) = -K(0) + C	
	C = In (18)	B1
	Let time of dog died t 11:00 am be t_1 — let t_1 be the time the dog died before 11:00 AM	
	At t,T =24.5 — At $t = t_1$ T = 24.5°c	
	In (24.5 – 21) = -Kt_1 +In(18)	
	In (3.5) = -Kt_1 + ln18 ______ (i)	m1
	At 12:00 pm(t_1+1) = t and T= 23.9	
	In (23.9 – 21) = -K(t_1+1) +In 18	
	In(2.9) = -Kt_1 + -K + In 18 ______(ii)	m1
	(i) – (ii) In(3.5) – In(2.9) = K K = In $\left(\frac{3.5}{2.9}\right)$	
	Substituting for K in (i)	
	In (3.5) = -In $\left(\frac{3.5}{2.9}\right)$ t_1 + In 18	m1
	In $\left(\frac{3.5}{2.9}\right)$$t_1$ =In 18 – In 3.5	
	ln 1.2069.t_1 = ln 5.1429	

$t_1 = \frac{\ln 5.1429}{\ln 1.2069}$

t_1 = 8.7082 hours **END**

$t_1 \simeq$ 8:42 minutes. A1

∴ The dog was knocked down at

11:00 AM
8:42 Hrs
2:18 Am A1

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